Sixth Annual Upper Peninsula
High School Math Challenge
Northern Michigan University (Marquette Co, MI)
Saturday 15 March 2014

NAME: _____ SOLUTION _____

TEAM: _____

SCHOOL: _____

SUDDEN DEATH

$$\frac{36}{91}$$

_____

answer

Put no work on this side of the paper. Write the answer and only the answer in the space above. Put all work on the other side of the sheet.

Amanda, Bertha, and Caitlyn are playing a dice game.

Amanda rolls 2 standard dice. If she rolls a 7 she wins; otherwise, she passes the dice to Bertha.

If Bertha rolls a 7, she wins; otherwise, she passes the dice to Caitlyn.

If Caitlyn rolls a 7, she wins; otherwise, she passes the dice back to Amanda, and so forth.

This game continues until someone rolls a 7. That person is declared the winner.

What is the probability that Amanda will win? Express your answer as a reduced fraction.

$7 \Rightarrow$
$1 \& 6$
$2 \& 5$ 6 ways out of 36,   probability of $\frac{1}{6}$
$3 \& 4$
$4 \& 3$
$5 \& 2$
$6 \& 1$

Amanda wins if

A   B

7 $\frac{1}{6}$

A   B   C   A

not 7   not 7   not 7   7 $\left(\frac{5}{6}\right)^3 \left(\frac{1}{6}\right)$

A   B   C   A   B   C   A

not 7 not 7 not 7 not 7 not 7 not 7 7 $\left(\frac{5}{6}\right)^6 \left(\frac{1}{6}\right)$

AND SO ON.

$\frac{1}{6} + \frac{1}{6}\left(\frac{5}{6}\right)^3 + \frac{1}{6}\left(\frac{5}{6}\right)^6 + \dots$

Sum of infinite geometric series is $\frac{9}{1-r}$

$\dfrac{\frac{1}{6}}{1-\left(\frac{5}{6}\right)^3} = \dfrac{\frac{1}{6}}{\frac{216-125}{216}} = \dfrac{\frac{1}{6}}{\frac{91}{216}} = \frac{1}{6} \cdot \frac{216}{91} = \frac{36}{91}$